# Lesson 2. The Shortest Path Problem, cont.

## 1 Examples

**Example 1.** The Dijkstra Brewing Company is planning production of its new limited run beer, Primal Pilsner. The company must supply 30 batches in the next quarter, then 25, 10, and 35 in successive quarters. Each quarter in which the company produces the beer requires a factory setup cost of $100,000. Each batch of beer costs $3,000 to produce. Batches can be held in inventory, but due to refrigeration requirements, the cost is a high $5,000 per batch per quarter. The company wants to find a production plan that minimizes its total cost. Formuate this problem as a shortest path problem.

**Example 2.** Beverly owns a vacation home in Cape Fulkerson that she wishes to rent for the summer season (May 1 to September 1). She has solicited bids from eight potential renters:

| Renter | Rental start date | Rental end date | Amount of bid ($) |
|---|---|---|---|
| 1 | May 1 | June 1 | 1800 |
| 2 | May 1 | July 1 | 3400 |
| 3 | June 1 | July 1 | 2000 |
| 4 | June 1 | August 1 | 4000 |
| 5 | June 1 | September 1 | 4800 |
| 6 | July 1 | August 1 | 1600 |
| 7 | July 1 | September 1 | 3200 |
| 8 | August 1 | September 1 | 1400 |

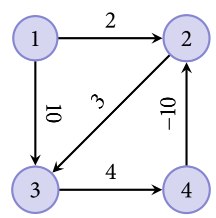
A rental starts at 15:00 on the start date, and ends at 12:00 on the end date. As a result, one rental can end and another rental can start on the same day. However, only one renter can occupy the vacation home at any time.

Beverly wants to identify a selection of bids that would maximize her total revenue. Formulate Beverly's problem as a shortest path problem.

## 2 Longest paths and negative cycles

- We saw in the previous example that formulating a shortest path problem with negative edge lengths often makes sense, especially when a problem is naturally formulated as a **longest path problem**
- This can sometimes be problematic!

**Example 3.** Find the shortest path from node 1 to node 4 in the following digraph:




- Remember that a path can visit each node at most once
- A **cycle** in a digraph is a path from a source node $s$ to a target node $t$ plus an arc $(t, s)$
- A **negative cycle** has negative total length
  - For example: $(2, 3), (3, 4), (4, 2)$ in the digraph above
- Negative cycles make things complicated: if we traverse a negative cycle, we can reduce the cost of getting point A to point B infinitely
- Shortest path problems with negative cycles harder to solve
  - Standard shortest path algorithms fail when the digraph has a negative cycle
- Having a negative cycle in your shortest path problem might indicate (i) your problem will be hard to solve, or (ii) there is a mistake in your formulation!